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## 8-6 Study Guide and Intervention *(continued)*

### *Solving Rational Equations and Inequalities*

**Solve Rational Inequalities** To solve a rational inequality, complete the following steps.

| | |
|---|---|
| **Step 1** | State the excluded values. |
| **Step 2** | Solve the related equation. |
| **Step 3** | Use the values from steps 1 and 2 to divide the number line into regions. Test a value in each region to see which regions satisfy the original inequality. |

**Example** Solve $\frac{2}{3n} + \frac{4}{5n} \leq \frac{2}{3}$.

**Step 1** The value of 0 is excluded since this value would result in a denominator of 0.

**Step 2** Solve the related equation.

$$\frac{2}{3n} + \frac{4}{5n} = \frac{2}{3}$$ Related equation

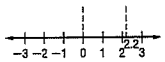
$$15n\left(\frac{2}{3n} + \frac{4}{5n}\right) = 15n\left(\frac{2}{3}\right)$$ Multiply each side by $15n$.

$$10 + 12 = 10n$$ Simplify.

$$22 = 10n$$ Simplify.

$$2.2 = n$$ Simplify.

**Step 3** Draw a number with vertical lines at the excluded value and the solution to the equation.




Test $n = -1$. $-\frac{2}{3} + \left(-\frac{4}{5}\right) \leq \frac{2}{3}$ is true.

Test $n = 1$. $\frac{2}{3} + \frac{4}{5} \leq \frac{2}{3}$ is *not* true.

Test $n = 3$. $\frac{2}{9} + \frac{4}{15} \leq \frac{2}{3}$ is true.

The solution is $n < 0$ or $n \geq 2.2$.

**Exercises**

**Solve each inequality.**

1. $\frac{3}{a+1} \geq 3$ $-1 < a \leq 0$

2. $\frac{1}{x} \geq 4x$ $x \leq -\frac{1}{2}$ or $0 < x \leq \frac{1}{2}$

3. $\frac{1}{2p} + \frac{4}{5p} > \frac{2}{3}$ $0 < p < \frac{39}{20}$

4. $\frac{3}{2x} - \frac{2}{x} > \frac{1}{4}$ $-2 < x < 0$

5. $\frac{4}{x-1} + \frac{5}{x} < 2$ $x < 0$ or $\frac{1}{2} < x < 1$ or $x > 5$

6. $\frac{3}{x^2-1} + 1 > \frac{2}{x-1}$ $x < -1$ or $0 < x < 1$ or $x > 2$





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## 8-6 Skills Practice

### *Solving Rational Equations and Inequalities*

**Solve each equation or inequality. Check your solutions.**

1. $\frac{x}{x-1} = \frac{1}{2}$ −1

2. $2 = \frac{4}{n} + \frac{1}{3}$ $\frac{12}{5}$

3. $\frac{9}{3x} = \frac{-6}{2}$ −1

4. $3 - z = \frac{2}{z}$ 1, 2

5. $\frac{2}{d+1} = \frac{1}{d-2}$ 5

6. $\frac{s-3}{5} = \frac{8}{s}$ −5, 8

7. $\frac{2x+3}{x+1} = \frac{3}{2}$ −3

8. $-\frac{12}{y} = y - 7$ 3, 4

9. $\frac{x-2}{x+4} = \frac{x+1}{x+10}$ 8

10. $\frac{3}{k} - \frac{4}{3k} > 0$ $k > 0$

11. $2 - \frac{3}{v} < \frac{5}{v}$ $0 < v < 4$

12. $n + \frac{3}{n} < \frac{12}{n}$ $n < -3$ or $0 < n < 3$

13. $\frac{1}{2m} - \frac{3}{m} < -\frac{5}{2}$ $0 < m < 1$

14. $\frac{1}{2x} < \frac{2}{x} - 1$ $0 < x < \frac{3}{2}$

15. $\frac{15}{x} + \frac{9x-7}{x+2} = 9$ 3

16. $\frac{3b-2}{b+1} = 4 - \frac{b+2}{b-1}$ 4

17. $2 = \frac{5}{2q} + \frac{2q}{q+1}$ −5

18. $8 - \frac{4}{z} = \frac{8z-8}{z+2}$ $\frac{2}{5}$

19. $\frac{1}{n+3} + \frac{5}{n^2-9} = \frac{2}{n-3}$ −4

20. $\frac{1}{w+2} + \frac{1}{w-2} = \frac{4}{w^2-4}$ ∅

21. $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$ ∅

22. $\frac{12s+19}{s^2+7s+12} - \frac{3}{s+3} = \frac{5}{s+4}$ 2

23. $\frac{2e}{e^2-4} + \frac{1}{e-2} = \frac{2}{e+2}$ −6

24. $\frac{8}{t^2-9} + \frac{4}{t+3} = \frac{2}{t-3}$ 5











Answers

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# 8-6 Practice

## Solving Rational Equations and Inequalities

**Solve each equation or inequality. Check your solutions.**

1. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$ 16
2. $\frac{x}{x-1} - 1 = \frac{x}{2}$ $-1, 2$
3. $\frac{p+10}{p^2-2} = \frac{4}{p}$ $-\frac{2}{3}, 4$
4. $\frac{s}{s+2} + s = \frac{5s+8}{s+2}$ 4
5. $\frac{5}{y-5} = \frac{y}{y-5} - 1$ all reals except 5
6. $\frac{1}{3x-2} + \frac{5}{x} = 0$ $\frac{5}{8}$
7. $\frac{5}{t} < \frac{9}{2t+1}$ $t < -5$ or $-\frac{1}{2} < t < 0$
8. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$ $\frac{11}{5}$
9. $\frac{4}{w-2} = \frac{-1}{w+3}$ $-2$
10. $5 - \frac{3}{a} < \frac{7}{a}$ $0 < a < 2$
11. $\frac{4}{5x} + \frac{1}{10} < \frac{3}{2x}$ $0 < x < 7$
12. $8 + \frac{3}{y} > \frac{19}{y}$ $y < 0$ or $y > 2$
13. $\frac{4}{p} + \frac{1}{3p} < \frac{1}{5}$ $p < 0$ or $p > \frac{65}{3}$
14. $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$ $\varnothing$
15. $g + \frac{g}{g-2} = \frac{2}{g-2}$ $-1$
16. $b + \frac{2b}{b-1} = 1 - \frac{b-3}{b-1}$ $-2$
17. $\frac{1}{n+2} + \frac{1}{n-2} = \frac{3}{n^2-4}$ $\frac{3}{2}$
18. $\frac{c+1}{c-3} = 4 - \frac{12}{c^2-2c-3}$ $-\frac{5}{3}, 5$
19. $\frac{3}{k-3} + \frac{4}{k-4} = \frac{25}{k^2-7k+12}$ 7
20. $\frac{4v}{v-1} - \frac{5v}{v-2} = \frac{2}{v^2-3v+2}$ $-1, -2$
21. $\frac{y}{y+2} + \frac{7}{y-5} = \frac{14}{y^2-3y-10}$ 0
22. $\frac{x^2+4}{x^2-4} + \frac{x}{2-x} = \frac{2}{x+2}$ $\varnothing$
23. $\frac{r}{r+4} + \frac{4}{r-4} = \frac{r^2+16}{r^2-16}$ all reals except $-4$ and 4
24. $3 = \frac{6a-1}{2a+7} + \frac{22}{a+5}$ $-2$

27. **BASKETBALL** Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next $x$ free throws in a row, the function $f(x) = \frac{9+x}{19+x}$ represents Kiana's new ratio of free throws made. How many successful free throws in a row will raise Kiana's percent made to 60%? Is this a reasonable answer? Explain. **6; Sample answer: It is a reasonable answer. She will have made 15 out of 25 free throws, which is equivalent to 60%.**

28. **OPTICS** The lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relates the distance $p$ of an object from a lens, the distance $q$ of the image of the object from the lens, and the focal length $f$ of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters? Is this a reasonable answer? Explain. **20 cm; Sample answer: It is a reasonable answer, since $\frac{1}{20} + \frac{1}{5} = \frac{1}{4}$.**





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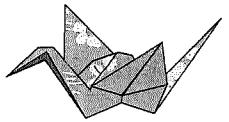
# 8-6 Word Problem Practice

## Solving Rational Equations and Inequalities

1. **HEIGHT** Serena can be described as being 8 inches shorter than her sister Malia, or as being 12.5% shorter than Malia. In other words, $\frac{8}{H+8} = \frac{1}{8}$, where $H$ is Serena's height in inches. How tall is Serena?

   **56 inches**

2. **CRANES** For a wedding, Paula wants to fold 1000 origami cranes.



   She does not want to make anyone fold more than 15 cranes. In other words, if $N$ is the number of people enlisted to fold cranes, Paula wants $\frac{1000}{N} \leq 15$. What is the minimum number of people that will satisfy this inequality?

   **67**

3. **RENTAL** Carlos and his friends rent a car. They split the $200 rental fee evenly. Carlos, together with just two of his friends, decide to rent a portable DVD player as well, and split the $30 rental fee for the DVD player evenly among themselves. Carlos ends up spending $50 for these rentals. Write an equation involving $N$, the number of friends Carlos has, using this information. Solve the equation for $N$.

   $\frac{200}{N+1} + \frac{30}{3} = 50$; $N = 4$

4. **PROJECTILES** A projectile target is launched into the air. A rocket interceptor is fired at the target. The ratio of the altitude of the rocket to the altitude of the projectile $t$ seconds after the launch of the rocket is given by the formula $\frac{333t}{-32t^2 + 420t + 27}$. At what time are the target and interceptor at the same altitude?

   **at $t = 3$ seconds**

**FLIGHT TIME For Exercises 5 and 6, use the following information.**

The distance between New York City and Los Angeles is about 2500 miles. Let $S$ be the airspeed of a jet. The wind speed is 100 miles per hour. Because of the wind, it takes longer to fly one way than the other.

5. Write an equation for $S$ if it takes 2 hours and 5 minutes longer to fly between New York and Los Angeles against the wind versus flying with the wind.

   $\frac{2500}{S-100} - \frac{2500}{S+100} = 2\frac{1}{12}$

6. Solve the equation you wrote in Exercise 5 for $S$.

   **500 mph**

7. Write an equation and find how much longer to fly between New York and Los Angeles if the wind speed increases to 150 miles per hour and the airspeed of the jet is 525 miles per hour.

   $\frac{2500}{525-150} - \frac{2500}{525+150} = x$;

   $x = \frac{400}{135} = 2.96$ h

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# 8-6 Enrichment

## Oblique Asymptotes

The graph of $y = ax + b$, where $a \neq 0$, is called an oblique asymptote of $y = f(x)$ if the graph of $f$ comes closer and closer to the line as $x \to \infty$ or $x \to -\infty$. $\infty$ is the mathematical symbol for **infinity**, which means *endless*.

For $f(x) = 3x + 4 + \frac{2}{x}$, $y = 3x + 4$ is an oblique asymptote because $f(x) - 3x - 4 = \frac{2}{x}$, and $\frac{2}{x} \to 0$ as $x \to \infty$ or $-\infty$. In other words, as $|x|$ increases, the value of $\frac{2}{x}$ gets smaller and smaller approaching 0.

**Example** **Find the oblique asymptote for $f(x) = \frac{x^2 + 8x + 15}{x + 2}$.**

$-2$ | 1 8 15 Use synthetic division.
$\quad$ | $\quad$ −2 −12
$\quad$ | 1 6 3

$y = \frac{x^2 + 8x + 15}{x + 2} = x + 6 + \frac{3}{x + 2}$

As $|x|$ increases, the value of $\frac{3}{x + 2}$ gets smaller. In other words, since $\frac{3}{x + 2} \to 0$ as $x \to \infty$ or $x \to -\infty$, $y = x + 6$ is an oblique asymptote.

**Use synthetic division to find the oblique asymptote for each function.**

1. $y = \frac{8x^2 - 4x + 11}{x + 5}$ $y = 8x - 44$

2. $y = \frac{x^2 + 3x - 15}{x - 2}$ $y = x + 5$

3. $y = \frac{x^2 - 2x - 18}{x - 3}$ $y = x + 1$

4. $y = \frac{ax^2 + bx + c}{x - d}$ $y = ax + b + ad$

5. $y = \frac{ax^2 + bx + c}{x + d}$ $y = ax + b - ad$









Answers