NAME ______________________ DATE ____________ PERIOD ____

9-6 Study Guide and Intervention

Exponential Growth and Decay

Exponential Decay Depreciation of value and radioactive decay are examples of **exponential decay**. When a quantity decreases by a fixed percent each time period, the amount of the quantity after t time periods is given by $y = a(1 - r)^t$, where a is the initial amount and r is the percent decrease expressed as a decimal.

Another exponential decay model often used by scientists is $y = ae^{-kt}$, where k is a constant.

Example **CONSUMER PRICES As technology advances, the price of many technological devices such as scientific calculators and camcorders goes down. One brand of hand-held organizer sells for $89.**

a. If its price decreases by 6% per year, how much will it cost after 5 years?

Use the exponential decay model with initial amount $89, percent decrease 0.06, and time 5 years.

$y = a(1 - r)^t$ Exponential decay formula

$y = 89(1 - 0.06)^5$ $a = 89$, $r = 0.06$, $t = 5$

$y = \$65.32$

After 5 years the price will be $65.32.

b. After how many years will its price be $50?

To find when the price will be $50, again use the exponential decay formula and solve for t.

$y = a(1 - r)^t$ Exponential decay formula

$50 = 89(1 - 0.06)^t$ $y = 50$, $a = 89$, $r = 0.06$

$\frac{50}{89} = (0.94)^t$ Divide each side by 89.

$\log\left(\frac{50}{89}\right) = \log (0.94)^t$ Property of Equality for Logarithms

$\log\left(\frac{50}{89}\right) = t \log 0.94$ Power Property

$t = \frac{\log\left(\frac{50}{89}\right)}{\log 0.94}$ Divide each side by log 0.94.

$t \approx 9.3$

The price will be $50 after about 9.3 years.

Exercises

1. **BUSINESS** A furniture store is closing out its business. Each week the owner lowers prices by 25%. After how many weeks will the sale price of a $500 item drop below $100?

6 weeks $100 = 500(1 - 0.25)^t$

CARBON DATING Use the formula $y = ae^{-0.00012t}$, where a is the initial amount of Carbon-14, t is the number of years ago the animal lived, and y is the remaining amount after t years.

2. How old is a fossil remain that has lost 95% of its Carbon-14? 24,964.4 years old

$5 = 100e^{-0.00012t}$

3. How old is a skeleton that has 95% of its Carbon-14 remaining?

427.5 years old

$95 = 100e^{-0.00012t}$

9-6 Study Guide and Intervention *(continued)*

Exponential Growth and Decay

Exponential Growth Population increase and growth of bacteria colonies are examples of **exponential growth**. When a quantity increases by a fixed percent each time period, the amount of that quantity after t time periods is given by $y = a(1 + r)^t$, where a is the initial amount and r is the percent increase (or rate of growth) expressed as a decimal.

Another exponential growth model often used by scientists is $y = ae^{kt}$, where k is a constant.

Example **A computer engineer is hired for a salary of $28,000. If she gets a 5% raise each year, after how many years will she be making $50,000 or more?**

Use the exponential growth model with $a = 28{,}000$, $y = 50{,}000$, and $r = 0.05$ and solve for t.

$y = a(1 + r)^t$	Exponential growth formula
$50{,}000 = 28{,}000(1 + 0.05)^t$	$y = 50{,}000$, $a = 28{,}000$, $r = 0.05$
$\frac{50}{28} = (1.05)^t$	Divide each side by 28,000.
$\log\left(\frac{50}{28}\right) = \log\,(1.05)^t$	Property of Equality of Logarithms
$\log\left(\frac{50}{28}\right) = t \log 1.05$	Power Property
$t = \dfrac{\log\left(\frac{50}{28}\right)}{\log 1.05}$	Divide each side by log 1.05.
$t \approx 11.9$ years	Use a calculator.

If raises are given annually, she will be making over $50,000 in 12 years.

Exercises

1. **BACTERIA GROWTH** A certain strain of bacteria grows from 40 to 326 in 120 minutes. Find k for the growth formula $y = ae^{kt}$, where t is in minutes. $326 = 40e^{k \cdot 120}$

   ≈ 0.0175

2. **INVESTMENT** Carl plans to invest $500 at 8.25% interest, compounded continuously. How long will it take for his money to triple? $1500 = 500(1 + 0.0825)^t$

   ≈ 13.9 years

3. **SCHOOL POPULATION** There are currently 850 students at the high school, which represents full capacity. The town plans an addition to house 400 more students. If the school population grows at 7.8% per year, in how many years will the new addition be full?

   $1250 = 850(1 + 0.078)^t$

   ≈ 5.1 years

4. **EXERCISE** Hugo begins a walking program by walking $\frac{1}{2}$ mile per day for one week. Each week thereafter he increases his mileage by 10%. After how many weeks is he walking more than 5 miles per day?

   $5 = (\frac{1}{2})(1 + 0.10)^t$

   ≈ 24.2 weeks

5. **VOCABULARY GROWTH** When Emily was 18 months old, she had a 10-word vocabulary. By the time she was 5 years old (60 months), her vocabulary was 2500 words. If her vocabulary increased at a constant percent per month, what was that increase?

   $2500 = 10(1 + r)^{42}$

   $\approx 14\%$