Synthetic Division and Application:

6-3 p. 325-330

Let's look at how we learned to do long division.

$$
\begin{array}{r}
x^2 - 4x - 12 \\
x - 4 \overline{)\, x^3 - 8x^2 + 4x - 9} \\
\underline{(-)\, x^3 - 4x^2} \\
-4x^2 + 4x \\
\underline{(-)\, -4x^2 + 16x} \\
-12x - 9 \\
\underline{(-)\, -12x + 48} \\
-57
\end{array}
$$

The quotient is $x^2 - 4x - 12$, and the remainder is -57.

Therefore $\dfrac{x^3 - 8x^2 + 4x - 9}{x - 4} = x^2 - 4x - 12 - \dfrac{57}{x - 4}$.

What do you notice about my variables??

Synthetic Division: $(5x^3 - 13x^2 + 10x - 8) \div (x - 2)$

$5x^3 - 13x^2 + 10x - 8$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$5 \quad -13 \quad 10 \quad -8$

$2 \mid \; 5 \quad -13 \quad 10 \quad -8$

$\qquad\quad \nearrow 10 \quad \nearrow -6 \quad \nearrow 8$

$5 \quad -3 \quad 4 \quad 10$

$5x^2 - 3x + 4$

$(x^3 - 8x^2 + 4x - 9) \div (x - 4)$

$1 \quad -8 \quad 4 \quad -9$

$4 \mid \; 4 \quad -16 \quad -48$

$1 \quad -4 \quad -12 \mid -57$

$x^2 - 4x - 12 \; \frac{-57}{x-4}$

NAME _____ DATE _____ PERIOD ____

6-3 Study Guide and Intervention *(continued)*

Dividing Polynomials

Use Synthetic Division

Synthetic division	a procedure to divide a polynomial by a binomial using coefficients of the dividend and the value of r in the divisor $x - r$

Use synthetic division to find $(2x^3 - 5x^2 + 5x - 2) \div (x - 1)$.

Step 1	Write the terms of the dividend so that the degrees of the terms are in descending order. Then write just the coefficients.	$2x^3 - 5x^2 + 5x - 2$ 2 −5 5 −2
Step 2	Write the constant r of the divisor $x - r$ to the left. In this case, $r = 1$. Bring down the first coefficient, 2, as shown.	1 | 2 −5 5 −2 2
Step 3	Multiply the first coefficient by r, $1 \cdot 2 = 2$. Write their product under the second coefficient. Then add the product and the second coefficient: $-5 + 2 = -3$.	1 | 2 −5 5 −2 2 2 −3
Step 4	Multiply the sum, −3, by r: $-3 \cdot 1 = -3$. Write the product under the next coefficient and add: $5 + (-3) = 2$.	1 | 2 −5 5 −2 2 −3 2 −3 2
Step 5	Multiply the sum, 2, by r: $2 \cdot 1 = 2$. Write the product under the next coefficient and add: $-2 + 2 = 0$. The remainder is 0.	1 | 2 −5 5 −2 2 −3 2 2 −3 2 0

Thus, $(2x^3 - 5x^2 + 5x - 2) \div (x - 1) = 2x^2 - 3x + 2$.

Exercises

Simplify. (see answer key)

1. $(3x^3 - 7x^2 + 9x - 14) \div (x - 2)$
2. $(5x^3 + 7x^2 - x - 3) \div (x + 1)$
3. $(2x^3 + 3x^2 - 10x - 3) \div (x + 3)$
4. $(x^3 - 8x^2 + 19x - 9) \div (x - 4)$
5. $(2x^3 + 10x^2 + 9x + 38) \div (x + 5)$
6. $(3x^3 - 8x^2 + 16x - 1) \div (x - 1)$
7. $(x^3 - 9x^2 + 17x - 1) \div (x - 2)$
8. $(4x^3 - 25x^2 + 4x + 20) \div (x - 6)$
9. $(6x^3 + 28x^2 - 7x + 9) \div (x + 5)$
10. $(x^4 - 4x^3 + x^2 + 7x - 2) \div (x - 2)$
11. $(12x^4 + 20x^3 - 24x^2 + 20x + 35) \div (3x + 5)$

NAME _____ DATE _____ PERIOD ____

6-3 Word Problem Practice

Dividing Polynomials

1. **REMAINDERS** Jordan divided the polynomial $x^4 + x - 6$ into the polynomial $p(x)$ yesterday. Today his work is smudged and he cannot read $p(x)$ or most of his answer. The only part he could read was the remainder $x + 4$. His teacher wants him to find $p(-3)$. What is $p(-3)$?

1

2. **LONG DIVISION** Dana used long division to divide $x^4 + x^3 + x^2 + x + 1$ by $x + 2$. Her work is shown below with three numbers missing.

$$
\begin{array}{r}
x^3 - x^2 + 3x - 5 \\
x + 2 \overline{)\, x^4 + x^3 + x^2 + x + 1} \\
(-)\, x^4 + 2x^3 \\
\hline
-x^3 + A \\
(-)\, -x^3 - 2x^2 \\
\hline
3x^2 + x \\
(-)\, 3x^2 + B \\
\hline
-5x + 1 \\
(-)\, -5x - 10 \\
\hline
C
\end{array}
$$

What are A, B, and C?

$A = x^2$, $B = 6x$, $C = 11$

3. **AVERAGES** Shelby is a statistician. She has a list of $n + 1$ numbers and she needs to find their average. Two of the numbers are n^3 and 2. Each of the other $n - 1$ numbers are all equal to 1. What is the average of these numbers?

$\frac{n^3 + 2 + (n-1)(1)}{n+1}$

$n^2 - n + 2 - \frac{1}{n+1}$

$n^2 - n + 2$ R 1

$n+1 \overline{)\, n^3 + 0n^2 + 0n + 3}$

$-n^3 + n^2$

$-n^2 + 0n + 3$

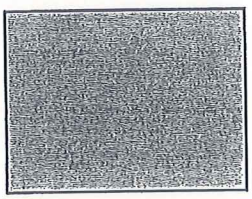
$(-)\ -n^2 - n$

$2n + 3$

$(-)\ 2n + 2$

1

4. **AREA** The area of a large rectangular sheet is $s^3 + 3s^2 + 4s + 1$ square inches.



If the length of the sheet is $s + 1$ inches, what is the width of the sheet?

$s^2 + 2s + 2 - \frac{1}{s+1}$

$-1 \mid 1 \quad 3 \quad 4 \quad 1$

$\quad\quad -1 \quad -2 \quad -2$

$1 \quad 2 \quad 2 \mid -1$

$s^2 + 2s + 2$

NUMBER THEORY For Exercises 5-6, use the following information.

Mr. Collins has his class working with bases and polynomials. He wrote on the board that the number 1111 in base B has the value $B^3 + B^2 + B + 1$. The class was then given the following questions to answer.

5. The number 11 in base B has the value $B + 1$. What is 1111 (in base B) divided by 11 (in base B)?

$B^2 + 1$

$B+1 \overline{)\, B^3 + B^2 + B + 1}$

$(-)\, B^3 + B^2$

$B + 1$

$(-)\ B + 1$

0

$B^2 + 1$

6. The number 111 in base B has the value $B^2 + B + 1$. What is 1111 (in base B) divided by 111 (in base B)?

B R 1

$B^2 + B + 1 \overline{)\, B^3 + B^2 + B + 1}$

$(-)\, B^3 + B^2 + B$

1

$B + \frac{1}{B^2 + B + 1}$

Exercises

Simplify.

1. $(3x^3 - 7x^2 + 9x - 14) \div (x - 2)$
   $3x^2 - x + 7$

2. $(5x^3 + 7x^2 - x - 3) \div (x + 1)$
   $5x^2 + 2x - 3$

3. $(2x^3 + 3x^2 - 10x - 9) \div (x + 3)$
   $2x^2 - 3x - 1$

4. $(x^3 - 8x^2 + 19x - 9) \div (x - 4)$
   $x^2 - 4x + 3 + \frac{3}{x - 4}$

5. $(2x^3 + 10x^2 + 9x + 38) \div (x + 5)$
   $2x^2 + 9 - \frac{7}{x + 5}$

6. $(3x^3 - 8x^2 + 16x - 1) \div (x - 1)$
   $3x^2 - 5x + 11 + \frac{10}{x - 1}$

7. $(x^3 - 9x^2 + 17x - 1) \div (x - 2)$
   $x^2 - 7x + 3 + \frac{5}{x - 2}$

8. $(4x^3 - 25x^2 + 4x + 20) \div (x - 6)$
   $4x^2 - x - 2 + \frac{8}{x - 6}$

9. $(6x^3 + 28x^2 - 7x + 9) \div (x + 5)$
   $6x^2 - 2x + 3 - \frac{6}{x + 5}$

10. $(x^4 - 4x^3 + x^2 + 7x - 2) \div (x - 2)$
    $x^3 - 2x^2 - 3x + 1$

11. $(12x^4 + 20x^3 - 24x^2 + 20x + 35) \div (3x + 5)$ $4x^3 - 8x + 20 + \frac{-65}{3x + 5}$

1. **REMAINDERS** Jordan divided the polynomial $x^4 + x - 6$ into the polynomial $p(x)$ yesterday. Today his work is smudged and he cannot read $p(x)$ or most of his answer. The only part he could read was the remainder $x + 4$. His teacher wants him to find $p(-3)$. What is $p(-3)$?
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-5x + 1 \\
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   What are A, B, and C?
   A is x^2; B is $6x$; C is 11.

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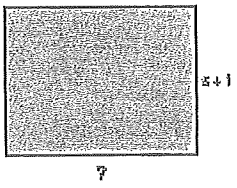

   If the length of the sheet is $s + 1$ inches, what is the width of the sheet?
   $s^2 + 2s + 2 - \frac{1}{s + 1}$ in.

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5. The number 11 in base B has the value $B + 1$. What is 1111 (in base B) divided by 11 (in base B)?
   $B^2 + 1$

6. The number 111 in base B has the value $B^2 + B + 1$. What is 1111 (in base B) divided by 111 (in base B)?
   $B + \frac{1}{B^2 + B + 1}$